Evidence on the Unobserved Processes that Generate Accruals: a Spectral Analysis

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Abstract

I study the underlying processes that contribute to observed accruals. First, I summarize Allen, Larson, and Sloan's (2013) findings on accrual reversals, earnings, and stock returns. They suggest that accruals consist of multiple underlying processes –each with different autocovariance properties, and point to one process with positive autocovariance, and one with negative autocovariance. I extend their endeavor by introducing a nonparametric statistical technique, spectral analysis, for studying the underlying processes that sum to an observed time series. My results suggest that (1) accrual periodicity length increases in operating cycle length, (2) there are multiple processes involved in accruals, (3) the most influential of these processes has a periodicity of less than a year, and (4) the periodicities of these processes are correlated with firm and earnings attributes.

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I. Introduction

This study aims to shed light on the unobservable processes that generate accruals. Casual observation suggests accruals are the result of multiple processes such as firm growth, temporary changes in working capital, earnings management, and seasonal variation. I provide statistical evidence that multiple distinct processes drive accruals. The most influential of these processes generates accruals that reverse less than a year after origination. I also provide evidence on how these processes correlate with firm characteristics (e.g., size, capital intensity) and earnings characteristics (e.g., persistence, relevance).

My study makes two significant contributions to the accounting literature. First, I extend our understanding of the processes that generate accruals. When researchers create measures of accruals quality, they implicitly assume that accruals can be divided into distinct processes ("good" and "bad" in many cases). If there are more processes than previously believed, then any measure of accrual quality will require a categorization of each process as "good" or "bad." We can only make these categorizations if we know what the processes are. Thus, understanding the processes that generate accruals is important for any endeavor that relies on a notion of accruals quality.

Second, I present spectral analysis, a statistical technique for investigating the unobservable processes that sum to an observed time series, to the accounting literature. This technique could be applied in further studies of accruals, but would also be appropriate for studying time series properties of cash-flows, account balances, or any of many other accounting time series.

This paper proceeds as follows: section II summarizes Allen, Larson, and Sloan; discusses their statistical methodology, and provides an alternative interpretation of their results. Section III provides a literature review and hypothesis development for the extension. Section IV introduces a new statistical method and discusses new tests involving that method. Section V concludes.

II. Allen, Larson, and Sloan (2013)

Allen, Larson, and Sloan (ALS here on) present three hypotheses. First, they hypothesize that multiple underlying processes drive accruals (H1). Specifically, they posit three processes: firm growth, temporary changes in work capital (TCWC here on) and error. They suggest that these processes have positive, negative, and negative autocorrelations, respectively.

Second, ALS hypothesize that, of these processes, error is primarily responsible for the lower persistence of the accruals component of earnings (H2). Lastly, they hypothesize that error is responsible for the negative correlation between accruals and stock returns (H3).

To test their hypotheses, the authors decompose accruals into growth, TCWC, and error components. They do so by regressing, for each industry, accruals on growth variables and TCWC variables:

$$Total\ Accruals_t = salesgrowth_t + employmentgrowth_t + CF_{t-1} + CF_t + CF_{t+1}$$

The authors refer to this regression throughout the paper as the MDD, or modified Dechow and Dichev, model. The portion of the MDD fitted value that comes from the growth variables is called MDDGROWTH, which the authors use as a proxy for accruals related to firm growth. Likewise, the portion of the fitted value that comes from the TCWC variables is called MDDMATCH. The difference between the fitted value and the observed accrual is called MDDERROR, which proxies for accruals related to error. The authors also use the term MDDGOOD, which refers to the total fitted value, i.e. MDDMATCH+MDDGROWTH.

After using the MDD model to decompose accruals, the authors provide two tests of H1. First, they compare the autocorrelation of MDDMATCH, MDDGROWTH, and MDDERROR. MDDMATCH is negatively autocorrelated, and MDDGROWTH is positively autocorrelated.

ALS take this as evidence that growth and TCWC are distinct processes driving accruals. Second, ALS perform a latent mixture regression of accruals on lagged accruals. The latent mixture regression splits observations into clusters, allowing for a different relationship between contemporaneous and lagged accruals within each cluster. Clusters are formed to maximize model AIC (Akaike Information Criterion). The results suggest at least three clusters are present, which ALS interpret as evidence that multiple distinct processes drive accruals.

ALS test H2 by regressing next year's income on contemporaneous cash flow, MDDMATCH, MDDGROWTH, and MDDERROR. MDDERROR has a lower coefficient than MDDGROWTH or MDDMATCH, suggesting that MDDERROR decreases the persistence of earnings.

Lastly, the authors test H3 by regressing next year's stock returns on contemporaneous cash flow, MDDMATCH, MDDGROWTH, and MDDERROR. Their results suggest that the negative relationship between accruals and stock return is driven primarily by MDDGROWTH, and only to a lesser extent by MDDERROR, counter to their original hypothesis.

II. Discussion and Alternative Interpretation

It is unclear whether ALS provide sufficient evidence to accept H1, that multiple processes drive accruals. Regarding H1, the MDD model and the latent mixture regression results do not necessarily imply that multiple processes drive accruals. The MDD model regresses accruals on growth variables (which are positively autocorrelated) and TCWC variables (which are negatively autocorrelated). The resulting fitted MDDGROWTH (MDDMATCH) values are positively (negatively) autocorrelated, which is the primary evidence that multiple processes drive accruals.

However, this autocorrelation structure of the fitted values could occur even if distinct processes with positive and negative autocorrelation did not drive accruals.

Consider the following example. Let X be a series of 100 observations, each distributed i.i.d. standard normal. In particular, note that X is a process which is not generated by multiple underlying processes. In fact, each element of X is independent of (not just uncorrelated with) every other element. Let Y be a series of 100 observations exhibiting positive autocorrelation of 0.5 (perhaps generated by an autoregressive model with auto-regressive constant 0.5 and error distributed standard normal). Let Z be a series of 100 observations exhibiting negative autocorrelation of -0.5.

Now, regress X on Y and Z. The portion of the fitted values contributed by Y will have positive autocorrelation. The portion of the fitted values contributed by Z will have negative autocorrelation. This is unsurprising because the partial fitted values due to Y are just the observed values of Y, which we know have positive autocorrelation, multiplied by a constant—the coefficient on Y. A similar argument applies for Z.

The point of this example is that any random series regressed on something positively autocorrelated will generate positively autocorrelated partial fitted values. Thus, MDDGROWTH's positive autocorrelation does not imply that an underlying positively autocorrelated process drives accruals. Rather, it provides evidence that firm growth is positively autocorrelated. A similar argument applies for negative autocorrelation and MDDMATCH.

Now I turn to the evidence provided in the latent mixture model (table four). Here, ALS introduce the latent mixture regression as an alternative method to decompose accruals. The latent class mixture model assumes each observation (firm-year) belongs to unobservable class C. The researcher chooses the number of classes. It is common to run a latent mixture model several times varying the number of classes, and choose the number of classes that maximizes the predictive value of the model. ALS settle on three classes. Thus, for our discussion, each firm-year observation can have C equal to 1, 2, or 3.

The latent mixture regression takes a dependent variable (accruals in this case), and one or more predictors (lagged accruals in this case). It divides the observations into three random clusters and regresses the dependent variable on the independent variables, by cluster. Then, it moves some observations to different clusters and reruns the regressions with the new clusters. The grouping scheme with greater likelihood is retained, and the process of regrouping and comparing likelihoods continues until the likelihood converges. There is no guarantee that the likelihood will converge to a global maximum. In fact, the statistics literature suggests that latent mixture regressions frequently converge only to a local maximum (BS Everitt 1996).

Assuming the latent mixture regression converges on a global maximum likelihood, it is unclear how to interpret the resulting clusters. Note that each firm may appear in different clusters in different years. Thus, if we say that each cluster represents a process driving accruals, we assume that each firm is driven only (or primarily) by one process in each year. We must also assume that the process is not consistent for firms across years and that each firm-year can be matched to its driving process (or cluster) using just two years of accrual information.

An alternative conceptualization of accruals processes allows each firm to have growth accruals, TCWC accruals, and error accruals in every period. If growth accruals are important for a firm in one period, they should probably be important in the next period too, since growth is positively autocorrelated and lasts longer than one period. If TCWC accruals are important for a firm in one period, they should probably be important in the next period too, since a firm's working capital volatility is relatively constant over time. This alternative conceptualization is incompatible with model implied by the latent cluster regression.

Given the difficulty and importance of decomposing accruals into components, more work is needed in this area.

IV. Literature Review and Hypotheses

Accruals can convey useful information to investors by matching cash flows to economic income, but they can also be used opportunistically to manipulate reported income (Dechow 1994). Accruals used to achieve the first goal are “good” accruals, and those used to achieve the second, “bad” accruals. Inspired by this distinction, much research, including ALS, has sought to understand the time series properties of accruals. I begin by testing ALS’s hypothesis H1:

H1: Multiple distinct processes drive accruals.

To understand the time series properties of accruals, it is helpful to review the time series properties of economic income. The qualities of economic income, along with other factors, drive the persistence of reported earnings. Different types of economic income will have different levels persistence, and thus generate accruals with differing levels of persistence. Along these lines, Chui, Li, and Radhakrishnan (2017) provide empirical evidence that slow reversing accruals are more persistent than fast reversing accruals, using operating cycle length to proxy for accrual reversal speed. Hence, their analysis hinges on the assumption that variability in operating cycle length correctly captures variability in accrual reversal speed. My analysis goes further by directly testing whether accrual reversal speed and operating cycle length are related. While Chui et al. find that operating cycle length (serving as a proxy for accrual lifespan) and persistence are positively correlated, I find that accrual lifespan and persistence are negatively correlated.

Moreover, our studies are conceptually different, because while they focus solely on persistence and accrual reversal speed, I focus on the properties of unobserved accrual processes more generally. Like them, however, I expect slower operating cycles to generate slower reversing accruals (H2).

H2: Longer operating cycles generate slower reversing accruals.

Another implication for the time series properties of economic income comes from Sticky Cost Theory. Collins, Punglalya, and Vijh explain:

> "Research evidence on sticky costs suggests that firms are slower to adjust inventory for sales declines than for sales increases (Banker and Chen 2006; Banker et al. 2015). So inventory accruals vary asymmetrically with respect to negative versus positive sales changes. Anderson, Lee, and Mashruwala (2015) report that labor costs and selling, general, and administrative expenses (SG&A) also exhibit asymmetric behavior with respect to positive and negative changes in sales. This implies that current working capital accruals like wages payable and accrued pension costs will behave asymmetrically with respect to sales declines versus sales increases."

Since inventory, wages payable, and accrued pension costs respond more slowly to sales decreases than sales increases, I expect that firms with higher propensity for losses will have slower reversing accruals than firms with few losses (H3).

H3: Firms with higher propensity for loss will have slower reversing accruals.

Now I turn to the time series properties of bad, or discretionary, accruals. One expectation is that the speed of noncurrent discretionary accrual reversals and the probability of meeting earnings targets will be inversely associated (Baber, Kang, and Li 2011). This is because reversals of discretionary accruals from prior periods constrain the initiation of discretionary accruals in the current period. The constraint on new discretionary accruals is more severe the faster the reversal speed of the previous accruals. Thus, faster reversing noncurrent discretionary accruals constrain the ability to meet or beat earnings forecasts. Baber et al. suggest the result applies only to noncurrent accruals because

> "…Managers are better able to control the reversal of non-current discretionary accruals than the reversal of current discretionary accruals. That is, reversals of prior non-current discretionary accruals are themselves discretionary relative to reversals of prior working capital discretionary accruals"

Overall, this line of reasoning suggests that faster reversing accruals, and especially faster reversing long-term accruals, increase the cost of earnings management. Thus, I expect accrual quality to decrease as accrual reversal speed increases. (H4).

H4: Accrual quality increases in the lifespan of noncurrent accruals.

Generally speaking, I expect long lasting accruals to carry more information content than short accruals. The information purpose of accruals is to match economic income to the period in which it was earned, even if cash flows did not occur in that period (Dechow 1994). Cash flows fail to capture this information about the timing of "earned" economic income when cash is received in a different period from when the income is earned. The importance of this failure increases as the time difference between the earning of the income and the associated cash flow increases. Thus, long lasting accruals, which match income to more temporally distant cash flows, should carry more information content. What's more, long lasting accruals are less likely to be used to beat short-term earnings targets, because the amount needed to beat the target is not known when the long-term accrual originates.

H5: Long lasting accruals have more information content, and are less likely to be discretionary, than short lasting accruals

Lastly, I expect that the shortest accruals, lasting about one fiscal quarter, are more common for large firms with high levels of liquidity. Large firms and firms with persistent earnings can engage in more transactions where cash is exchanged in adjacent periods, requiring the use of fast reversing accruals to match these transactions to their cash flows. Large firms may be more likely to engage in such "mismatched" transactions because they are less liquidity constrained (and so the timing of cash flows is less critical for their operations). For large firms, it is also possible that their size and resulting bureaucracy makes them "slow movers," who are unable to collect and disburse cash as quickly as smaller firms.[1]

H6: Fast reversing accruals, lasting only 1-2 quarters, will be more common for large firms and firms with persistent earnings than for smaller firms and firms with less persistent earnings.

IV. Spectral Analysis

In light of the need to identify the processes driving accruals, I introduce a statistical technique, new to the accounting literature, for investigating time series driven by multiple unobserved components. Spectral analysis, which is commonly used in engineering and the physical sciences, can help us understand the unobserved processes that contribute to accruals. Spectral analysis is an analysis of variance technique. It decomposes time series variance into portions explained by different frequencies, or periodicities, in the data (Percival and Walden, 1993). Insofar as different processes occur with different frequencies, it also decomposes time series variance into portions explained by different processes. The importance of each frequency is indicated by the "spectral density" of that frequency. For example, a time series of hourly temperature recordings over a decade would likely have a large spectral density for the frequency that implies 24-hour periodicity (since temperature fluctuates systematically with time of day) and

[1] Anecdotal evidence comes to mind here: a human resources mistake at a place like UT-Austin or the Mayo-Clinic (my work-place and a family member's, respectively) can cause a paycheck or reimbursement to be easily up to a month out of step with the underlying transaction. Meanwhile, a payroll error at my sister's summer-job restaurant can be rectified with a check written by the owner at the end of the day.

for the frequency that implies 1-year periodicities (since temperature fluctuates systematically across seasons).

A periodogram is a chart, for a given time series, that shows frequencies on the x-axis, and spectral density on the y-axis (Percival and Walden, 1993). I compute, on a firm-by-firm basis, a periodogram for the time series of quarterly accruals[2]. For each firm, I select the three frequencies with the most explanatory power, and label them as "the dominant frequencies" for the firm. The three dominant frequencies of the firm are labeled as "small," "medium," and "large" by frequency, not spectral density. Thus, "big dominant frequency" is the greatest frequency, of the three frequencies with the greatest spectral density. Figure one contains an example periodogram. I find that the dominant frequencies exhibit strong correlations with firm attributes such as size, operating cycle, intangibles intensity, cash flow variability, and incidence of losses. I also find that the dominant frequencies are strongly correlated with earnings attributes such as value relevance, persistence, and accruals quality.

Table one displays summary statistics for dominant frequencies, sorted by spectral density. The sample for this table, and the remaining spectral analysis tables that follow, is more restrictive than the sample for previous tables. I require 20 quarters of consecutive accruals data to compute the periodogram, which reduces my sample by about 30%. Within this sample, the average firm has its greatest spectral density at a frequency of .288, which implies a periodicity of 1/.288= 3.47 quarters. This is consistent with a large portion of accruals reversing within a year of origination. In untabulated tests, I find that the most (second most) dominant frequency explains over seven

[2] I use quarterly accruals, rather than annual, for the periodogram, because periodograms can only pick up periodicities of length greater than the minimum observation window. Thus, if I used annual data, I would not be able to detect periodicities of length less than one year. I also define Accruals as the difference between cash flows and income before extraordinary items, rather than the definition used by Sloan, because I want to include long term accruals like depreciation in my definition.

(four) times as much of the time series variance as would be expected under the null hypothesis of no periodicity.

Table two displays summary statistics for dominant frequencies, sorted by frequency length. The average firm has a smallest dominant frequency of .177 (implying a periodicity of 5.65 quarters), and a largest dominant frequency of .416 (implying a periodicity of 2.4 quarters). Paired t-tests (untabulated) confirm that firms' largest dominant frequencies are significantly different from their medium dominant frequencies, which are in turn significantly different from their smallest dominant frequencies. This provides evidence for hypothesis 1, which states that multiple processes with different reversal speeds are driving accruals.

I next investigate how these processes relate to firm and earnings attributes. Regarding firm attributes, I focus on firm size, operating cycle, intangibles intensity, capital intensity, cash flow variability, propensity for losses, and sales variability. Regarding earnings attributes, I focus on value relevance, smoothness, predictability, persistence, and accrual quality[3].

I loosely follow Francis, LaFond, Olsson, and Schipper in my choice and definition of these attributes (2003, FLOS here on). I compute size as log total assets. Sales variability is the standard deviation of quarterly sales, over a rolling 20 quarter window. Operating cycle is average inventory divided by quarterly COGS, plus average accounts receivable divided by total quarterly revenue, all multiplied by 91 to convert to days. Loss propensity is the proportion of the last 20 quarters having negative operating income before depreciation. Intangible intensity is quarterly R&D expense, plus annual advertising expense divided by 4, scaled by quarterly sales.[4]

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[3] Although I previously suggested that decomposing accruals into underlying processes is a prerequisite for defining accruals quality, I include a traditional measure of accrual quality here for comparison and reference purposes
[4] Quarterly advertising expense is not available on COMPUSTAT

Capital intensity is PPE scaled by total assets. Cash flow volatility is the standard deviation of quarterly operating cash flows over the past 20 quarters. Smoothness is the standard deviation of operating income before depreciation scaled by assets divided by the standard deviation of operating cash flows scaled by assets, over the last 20 quarters. Persistence is the coefficient from EPS regressed on last year's EPS, using a rolling 10-year window of observations. Predictability is the standard deviation of residuals from the same regression. Accrual quality is negative standard deviation of residuals from the MDD model, computed by firm with annual data over a 10-year rolling window. All variables are winsorized at the 1st and 99th percentiles. Table three provides summary statistics of firm and earnings attributes.

Tables five and six regress firm and earnings attributes on dominant accrual frequencies, sorted by frequency size. I cluster by firm and year because some variables are observed annually, some quarterly, and some only once per firm. Because periodicities are the inverses of frequencies, it can be confusing to interpret frequency coefficients in terms of periodicity lengths. For example, "smallest frequency decreases" is equivalent to "largest periodicity increases." Although periodicity lengths are more intuitive to think about, it is necessary to use frequencies in the regressions because periodicity lengths have nonconstant variance. To avoid confusion, table six summarizes the results of tables four and five in terms of periodicity length.

As expected, operating cycle length is strongly positively correlated with periodicity length, confirming H2 (second column of table 4). This is consistent with shorter operating cycles generating faster reversing accruals than longer operating cycles. Chui, Li, and Radhakrishnan (2017) assume this relationship in their analysis—but I empirically test it. I find that a 20%

increase in operating cycle length is associated with about a one fiscal quarter increase in accrual periodicity length, for the average firm[5].

Many variables are increasing (decreasing) in the length of the longest and shortest periodicities, but decreasing (increasing) in the length of the medium periodicity. This happens with smoothness, for example. The firms with the smoothest earnings have longer long and short periodicities and shorter medium periodicities. Likely, what's going on is that firms with strong 2-3 quarter length periodicities have very smooth earnings. Since 2-3 quarters is longer than the typical shortest periodicity but shorter than the typical medium periodicity, we would observe a positive relationship between smoothness and shortest periodicity, and a negative correlation between smoothness and medium periodicity.

See column six of table four for evidence on H3. H3 states that firms with higher propensity for loss will have slower reversing accruals because sticky costs cause accrued expenses to respond to losses and gains asymmetrically. Column six confirms firms with high propensity for loss have longer periodicities for their longest and shortest lasting accruals, consistent with H3. Interestingly, the length of medium lifespan accruals decreases in propensity for loss. Overall, I would characterize my results as mixed support for H3.

See the first row of table five for evidence on H4. H4 states that accrual quality decreases in the reversal speed of noncurrent accruals because fast reversing noncurrent accruals restrain managers' ability to initiate new discretionary accruals. By definition, noncurrent accruals last over a year. Thus, noncurrent accruals must have periodicity greater than 4 (frequency less than 0.25), so they are most likely to be captured by dom_freq_small. Dom_freq_small is negatively associated with value relevance and accrual quality, suggesting that faster reversing noncurrent

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[5] across frequency sizes

accruals are associated with lower accrual quality and value relevance, contrary to H4. My results are inconsistent with H4[6].

Turning to H5, which states that longer lasting accruals have higher information content, I find that longer longest periodicities are associated with more value relevance, smoother earnings, and greater accruals quality, even though they are also associated with longer operating cycles, higher intangible intensity, higher cash flow variability, and higher sales volatility. This is consistent with slow reversing accruals correctly matching volatile cash flows to a smoother true economic performance. Moreover, the shorter shortest periodicities are associated with less value relevance, less smoothness, less predictability, and lower accrual even though they are also associated with less volatile cash flows, less volatile sales, smaller propensity for loss, and higher earnings persistence. This suggests that very fast reversing accruals contribute are less successful in matching income to corresponding cash flows, and carry less valuable information to investors than do longer lasting accruals. Overall, my results provide strong evidence for H5.

Evidence on H6 can be found in column one of table four and column three of table five. H6 states that fast reversing accruals, lasting only 1-2 quarters, will be more important for large firms and firms with persistent earnings, compared to other firms. I find that large firms and firms with more persistent earnings have short accruals that reverse more quickly than those of other firms. This finding is consistent with large and high liquidity firms engaging in more transactions where cash is exchanged in adjacent periods, requiring the use of fast reversing accruals to match those transactions to their corresponding cash flows.

[6] Note that this does necessarily not imply that my results are inconsistent with Baber et al., who show that noncurrent accrual reversal speed and probability of meeting earnings targets are inversely related: (1) The probability of meeting earnings does not perfectly map onto accrual quality. (2) Baber et al. find a relationship between reversal speed and probability of meeting or beating earnings, holding accrual magnitude constant. My analysis does not hold accrual magnitude constant.

Lastly, and outside the scope of my hypotheses, I find that firms with shorter longest periodicities are also larger and have more persistent earnings. If the longest periodicity represents slow reversing accruals (like depreciation and amortization), this suggests that large firms, and firms with persistent earnings, depreciate their assets more quickly. Perhaps larger firms not only have more assets, but also use those assets more intensively and must replace them more often[7]. Similarly, firms with persistent earnings may use their assets more consistently resulting in faster depreciation than firms with less persistent earnings, whose assets receive less use during down periods.

Overall, I find strong support for H1, H2, H5, and H6, mixed support for H3, and no support for H4. It appears that multiple processes drive accruals, operating cycle length is positively associated with accrual periodicity length, longer last accruals have higher information content, and large, liquid firms use more short term accruals than other firms.

V. CONCLUSION

In this paper, I study the underlying processes that sum to observed accruals. I begin by summarizing Allen, Larson, and Sloan's study of whether multiple processes drive accruals and I provide an alternative interpretation of their empirical results. Specifically, I suggest that the autocorrelation properties of the fitted MDD components could arise mechanically from the autocorrelation properties of the MDD explanatory variables. I also investigate the characterization of accruals implied by ALS's latent mixture regression, and propose an alternative characterization of accruals and accrual sub-processes.

[7] For example, maybe a small firm uses one machine for 16 hours a day to produce 100 widgets. But a firm bigger firm, which produces 300 widgets, uses two machines for 24 hours a day. The bigger firm uses more machines, and it depreciates its machines faster.

Lastly, I perform a spectral analysis of accruals. Spectral analysis is an analysis of variance technique, new to the accounting literature, which is well suited for studying time series driven by multiple unobserved processes. My results suggest that there are multiple processes that sum to accruals, and that the most important of these processes occurs with high frequency (more than once a year). Lastly, I find that the periodicity lengths of these processes are strongly correlated with firm attributes, such as size, operating cycle length, and cash flow volatility; as well as with earnings attributes, such as value relevance, smoothness, and persistence.

Figure 1: Periodogram Example

Here is an example of a periodogram generated by the following simple time series of length 40:

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 [1]  0.0 -0.9 -1.8  1.3  0.4 -0.5 -1.6  1.3  0.2 -0.9 -2.0  1.1  0.2 -0.7 -1.6  1.5  0.4 -0.7 -1.8  1.1
[21]  0.0 -0.9 -1.8  1.3  0.4 -0.5 -1.6  1.3  0.2 -0.9 -2.0  1.1  0.2 -0.7 -1.6  1.5  0.4 -0.7 -1.8  1.1
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For example purposes, I manufactured this time series to have periodicities of length 4 and 10, by summing the sequences below. The sequences below can easily be observed to have periodicities of length 4 and 10. The periodogram shows frequencies on the x-axis, and spectral densities on the y-axis. The peak at frequency=0.1 implies a periodicity of length 1/0.1=10, and the peak at frequency=0.25 implies a periodicity of length 1/.25=4. For this series, maxfreq would be 0.25, maxfreq_2 would be 0.5, and maxfreq_3 would be 0.1. Sorting by size, dom_freq_big would be0 .5, big_freq_med would be 0.25, and big_freq_small would be 0.1. The high spectral density at frequency= 0.5 occurs because the first sequence below, which is formed by repeating (0,-1,-2,1), can actually be further decomposed as the sum of two repeating sequences: (0,1,0,1) which has periodicity of length 2, and (0,-2,-2,0), which has periodicity length 4. This example highlights the ability of the periodogram to pick up periodicities in data that are not immediately obvious to the naked eye.

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 [1]  0 -1 -2  1  0 -1 -2  1  0 -1 -2  1  0 -1 -2  1  0 -1 -2  1  0 -1 -2  1  0 -1 -2  1  0 -1 -2  1  0 -1
[35] -2  1  0 -1 -2  1
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```
 [1] 0.0 0.1 0.2 0.3 0.4 0.5 0.4 0.3 0.2 0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.4 0.3 0.2 0.1 0.0 0.1 0.2 0.3 0.4
[26] 0.5 0.4 0.3 0.2 0.1 0.0 0.1 0.2 0.3 0.4 0.5 0.4 0.3 0.2 0.1
```

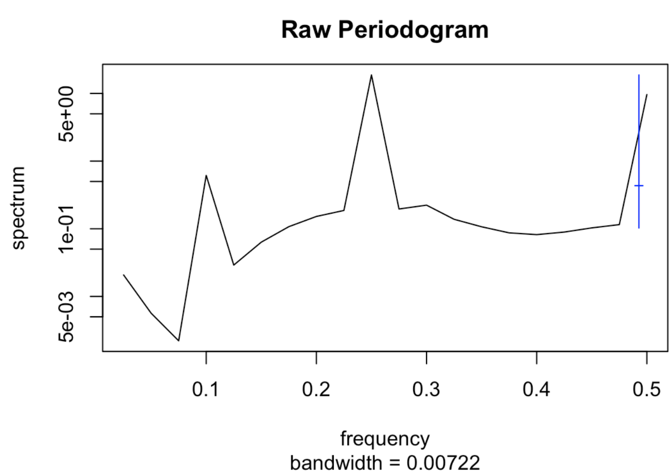

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Table 1: Summary Stats by Spectral Density

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
max_freq	0.288	0.128	0.250	0.250	0.333
max2_freq	0.308	0.150	0.229	0.259	0.500
max3_freq	0.264	0.135	0.167	0.250	0.338

Max_freq is the most powerful frequency for a given firm, max2_freq is the second most powerful frequency for a given firm, etc. The max_freq for each firm is calculated using the entire sample period of accruals data available for that firm. Frequencies can be converted to periodicities by dividing into one.

Table 2: Summary Stats by Frequency Length

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
dom_freq_small	0.177	0.086	0.102	0.222	0.250
dom_freq_med	0.265	0.096	0.244	0.250	0.273
dom_freq_big	0.416	0.112	0.300	0.500	0.500

Dom_freq_small is the smallest of the three most powerful frequencies for a given firm, dom_freq_big is the largest of the three most powerful frequencies for a given firm, etc.

Table 3: Summary Stats of Firm and Earnings Characteristics

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
log assets	5.266	2.040	3.754	5.147	6.657
op cycle	4.699	0.861	4.276	4.760	5.174
intangible intensity	0.185	0.433	0.033	0.082	0.174
capital intensity	0.289	0.227	0.108	0.225	0.414
CFO var	0.062	0.052	0.033	0.048	0.071
neg earn	0.151	0.272	0.000	0.000	0.150
SALES var	0.067	0.058	0.029	0.050	0.084
relevance	−0.372	0.241	−0.552	−0.344	−0.169
smoothness	0.329	0.229	0.174	0.268	0.415
predictibility	1.485	5.058	0.168	0.400	0.886
persistance	0.423	0.441	0.111	0.413	0.727
acc quality	0.016	0.012	0.008	0.013	0.021

Variable definitions are provided in the body of the paper. Summary statistics are produced from firm-quarter observations, and are winsorized at the 1st and 99th percentiles. Since some variables require data which is only available annually, standard deviations should be interpreted with caution.

Table 4: Firm qualities on frequencies

	Dependent variable:						
	size	op-cycle	intang-intensity	cap-intensity	CFO-var	losses	Sales-var
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
dom_freq_small	5.648^{***}	-0.658^{***}	-0.324^{**}	0.336^{***}	-0.024^{*}	-0.667^{***}	-0.034^{***}
	(0.417)	(0.160)	(0.137)	(0.043)	(0.013)	(0.082)	(0.012)
dom_freq_med	-2.365^{***}	-0.294^{*}	0.349^{**}	-0.118^{***}	0.034^{***}	0.456^{***}	0.004
	(0.404)	(0.151)	(0.144)	(0.045)	(0.010)	(0.065)	(0.010)
dom_freq_big	2.669^{***}	-0.542^{***}	0.195^{**}	0.303^{***}	-0.042^{***}	-0.150^{***}	-0.088^{***}
	(0.309)	(0.109)	(0.085)	(0.031)	(0.008)	(0.044)	(0.009)
Constant	3.747^{***}	5.119^{***}	0.070^{***}	0.131^{***}	0.077^{***}	0.217^{***}	0.110^{***}
	(0.153)	(0.043)	(0.025)	(0.012)	(0.004)	(0.018)	(0.005)

Note: *p<0.1; **p<0.05; ***p<0.01

Data are quarterly firm level observations. Since some variables are measured annually, errors are clustered by firm and fiscal-year.

Table 5: Earnings qualities on frequencies

	Dependent variable:				
	VR	smooth	pers	pred	acc quality
	(1)	(2)	(3)	(4)	(5)
dom_freq_small	-0.308^{***}	-0.526^{***}	0.458^{***}	-3.757	-0.029^{***}
	(0.051)	(0.052)	(0.091)	(2.335)	(0.006)
dom_freq_med	0.005	0.268^{***}	0.036	2.615	-0.004
	(0.050)	(0.044)	(0.086)	(1.634)	(0.005)
dom_freq_big	-0.224^{***}	-0.203^{***}	0.482^{***}	-3.892^{**}	-0.019^{***}
	(0.039)	(0.034)	(0.069)	(1.530)	(0.004)
Constant	-0.217^{***}	0.442^{***}	0.120^{***}	3.219^{***}	0.031^{***}
	(0.014)	(0.014)	(0.026)	(0.725)	(0.002)

Note: *p<0.1; **p<0.05; ***p<0.01

Data are quarterly firm level observations. Since some variables are measured annually, errors are clustered by firm and fiscal-year.

Table 6: Summary of relationships between periodicities, frequencies, and firm and earnings attributes

	longest periodicity	medium periodicity	shortest periodicity
	increases	increases	increases
	smallest frequency	medium frequency	largest frequency
	decreases	decreases	decreases
size	decreases	increases	decreases
operating cycle	increases	increases	increases
intangible intensity	increases	decreases	decreases
capital intensity	decreases	increases	decreases
CF volatility	increases	decreases	increases
propensity for losses	increases	decreases	increases
Sales Volatility	increases	insignificant	increases
relevance	increases	insignificant	increases
smoothness	increases	decreases	increases
persistance	decreases	insignificant	decreases
predictibility	insignificant	insignificant	increases
accrual quality	increases	insignificant	increases